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Note on the numerical solution for unsteady viscous flow past a circular cylinder

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The starting flow of a viscous fluid past a circular cylinder at Reynolds numbers 40 and 100 has been obtained by a numerical method. The method used is that developed by Payne (1957) but it has been extended here to cover a larger time interval.

At Reynolds number 40 Payne's result for the drag coefficient at time $t = 6$ is in reasonable agreement with Kawaguti's (1953) result for the steady case but if Payne's computation is extended to time $t \approx 24$, the result is in better agreement with Apelt's (1961) result for the steady case. Also, a further investigation into the case $R = 100$ shows that Payne's mesh size is too crude. Similar observations can be made concerning the size of the standing vortices downstream of the circular cylinder and how they grow in time.

1. Computations

For Reynolds number 40 the following computations were performed (the notation and computational method is that of Payne 1957): (i) the space mesh size $\Delta = \pi/15 = 0{\cdot}20944$, so that there were 30 mesh points round the cylinder at intervals of 12 degrees, and $\Delta t = 0{\cdot}1$, i.e. Payne's computation; (ii) $\Delta = \pi/15$, and initially $\Delta t = 0{\cdot}05$; Δt was doubled on occasions—the criteria for doubling being that the same value of the drag, to within a prescribed value, could be obtained with the larger mesh size; (iii) $\Delta = \pi/20$ with a variable Δt as in (ii), $\Delta t = 0{\cdot}025$ initially; (iv) $\Delta = \pi/25$ with a variable Δt as in (ii), $\Delta t = 0{\cdot}01$ initially; (v) the starting solution for small time ($t \ll 1$) as obtained by Goldstein & Rosenhead (1936) was taken and then a variable mesh size Δ was used starting with $\Delta = \pi/120$ and finishing up with $\Delta = \pi/15$.

For Reynolds number 100 the following computations were performed: (*a*) $\Delta = \pi/15$ using a variable Δt as in (ii) with $\Delta t = 0{\cdot}1$ initially, these results coincide with Payne's calculations; (*b*) $\Delta = \pi/20$ using a variable Δt as in (ii) with $\Delta t = 0{\cdot}05$ initially.

2. Results

The variation of the drag coefficient, C_D, for t in the range $0 \leqslant t \leqslant 4$ is plotted in figure 1 for Reynolds number 40. This shows how the value of C_D depends critically on the mesh size near $t = 0$ which is to be expected since at small times

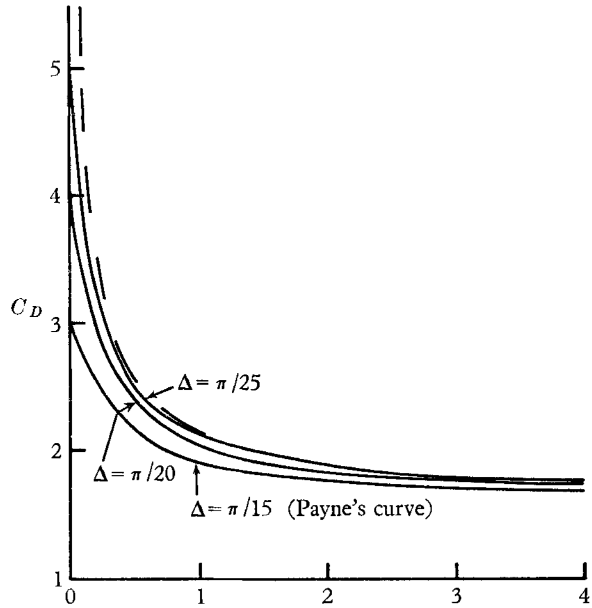

FIGURE 1. The variation of the drag coefficient with time for the starting flow past a circular cylinder for $R = 40$. – – – is for computation (v).

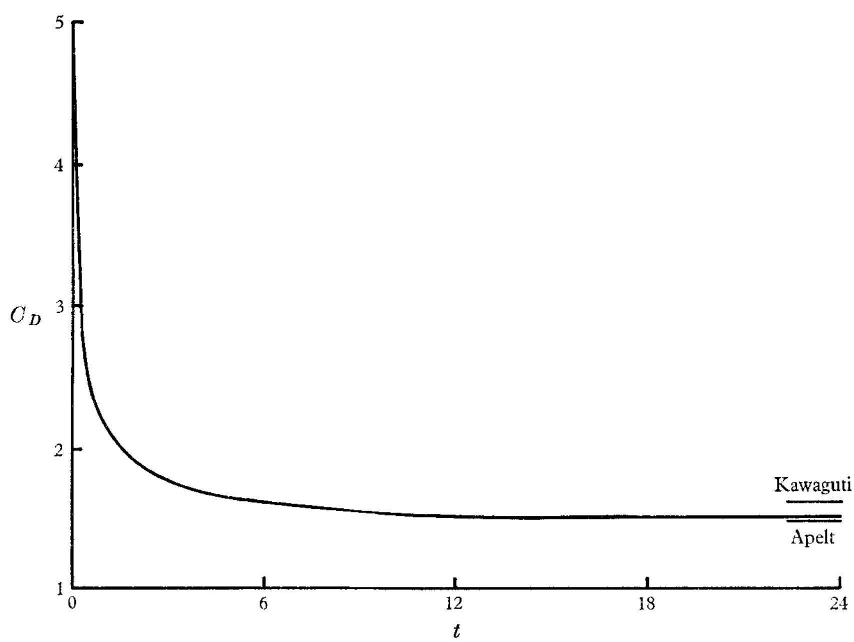

FIGURE 2. The variation of the drag coefficient with time for $R = 40$ and computation (v).

the boundary layer is very thin and hence better results are obtained with the smaller mesh points as there are more mesh points in the boundary layer. Figure 2 shows the result of computation (v) for t in the range 0 to 24; in the time interval $4 < t < 24$ the other computations are within about 1 % of this value of C_D and are therefore not presented. It is seen from figure 1, even at small times, as t increases the values of C_D obtained by using the different mesh sizes are tending towards the same value. In fact (i), (ii), (iii) and (v) all give at $t \approx 24$ the drag coefficient to be $1{\cdot}51 \pm 0{\cdot}01$ and the value appears to have settled down to this steady value. Computation (iv) has not been carried out to times $O(24)$ because of the large amount of computing time required.

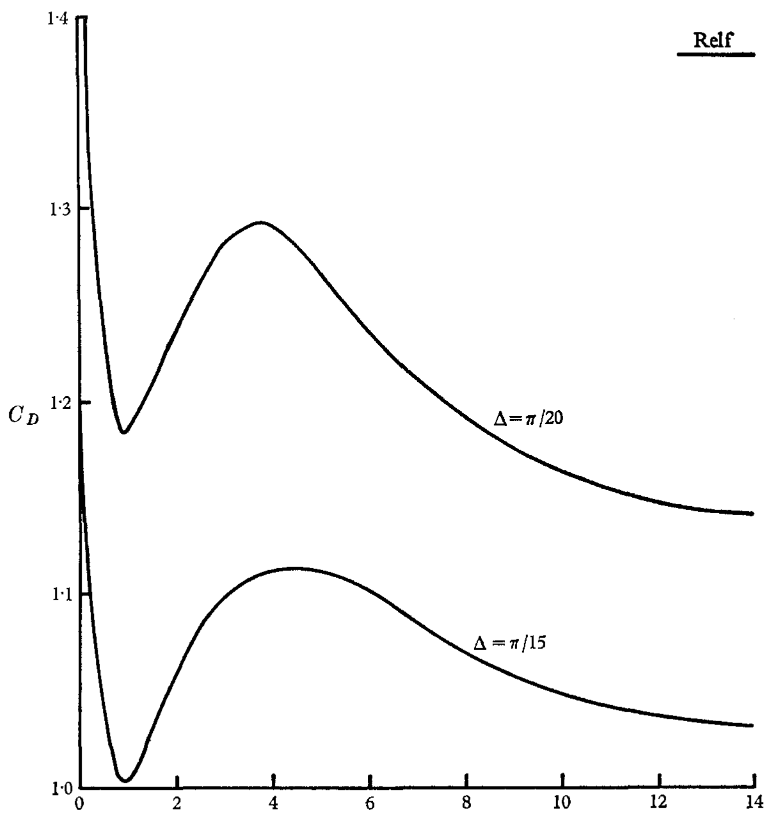

FIGURE 3. The variation of the drag coefficient with time for $R = 100$.

For Reynolds number 100 the variation of the drag coefficient with time is plotted in figure 3. It is seen that with neither mesh size does the drag coefficient approach the steady value as obtained by Relf (see Thom 1933) but the smaller mesh size, as expected, gives a better approximation.

Figure 4 shows how the length of the pair of vortices downstream of the cylinder grows in time for Reynolds numbers of 40. It is seen that $2E/d$, where E is the length of the pair of vortices, appears to be approaching a constant value for each computation for sufficiently large times, but in no case is this the steady state value as derived by Kawaguti or Apelt. Again it is seen that the smaller mesh size gives a better approximation.

At time $t = 6$ the drag coefficients for computations (i) and (ii) are about 2 % above the value of 1·6177 calculated by Kawaguti for the steady flow, as noted by Payne, but the value of C_D at time $t \approx 24$ is in better agreement with the results of Apelt who predicted a value of $C_D = 1{\cdot}496$. The most likely reason for the discrepancy between Apelt's solution and the one obtained here, by considering the unsteady problem at large times, is that the mesh size is too crude. The

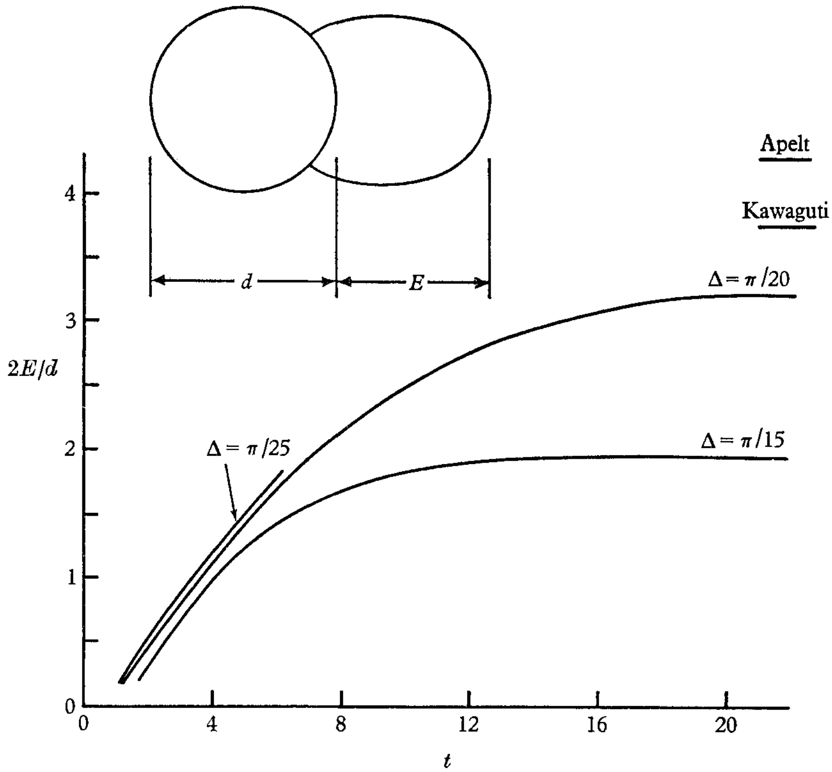

FIGURE 4. The variation of the length of the vortex pair attached to the cylinder with time. The values obtained by Apelt (1961) and Kawaguti (1953) for the steady flow at Reynolds number 40 are also shown.

values of vorticity are therefore not exact and since C_D is obtained by summing over all mesh points, which involves the multiplication of the vorticity and the distance from the plane of symmetry, y, a slight error in the vorticity at large values of y can have quite a marked effect on C_D.

The streamlines and lines of constant vorticity are very similar to those displayed by Payne in general and therefore are not presented here.

REFERENCES

APELT, C. J. 1961 *Aero. Res. Counc. (Lond.), Rept. & Mem.* no. 3175.
GOLDSTEIN, S. & ROSENHEAD, L. 1936 *Proc. Camb. Phil. Soc.* **32**, 392.
KAWAGUTI, M. 1953 *J. Phys. Soc. Japan*, **8**, 747.
PAYNE, R. B. 1957 *J. Fluid Mech.* **4**, 81.
THOM, A. 1933 *Proc. Roy. Soc.* A **141**, 651.